LESSON 6.3a

Introduction to Logarithms

Today you will:

- Learn what logarithms and logarithm functions are and how to evaluate them
- Learn about two special logarithms: common logs and natural logs
- Practice using English to describe math processes and equations

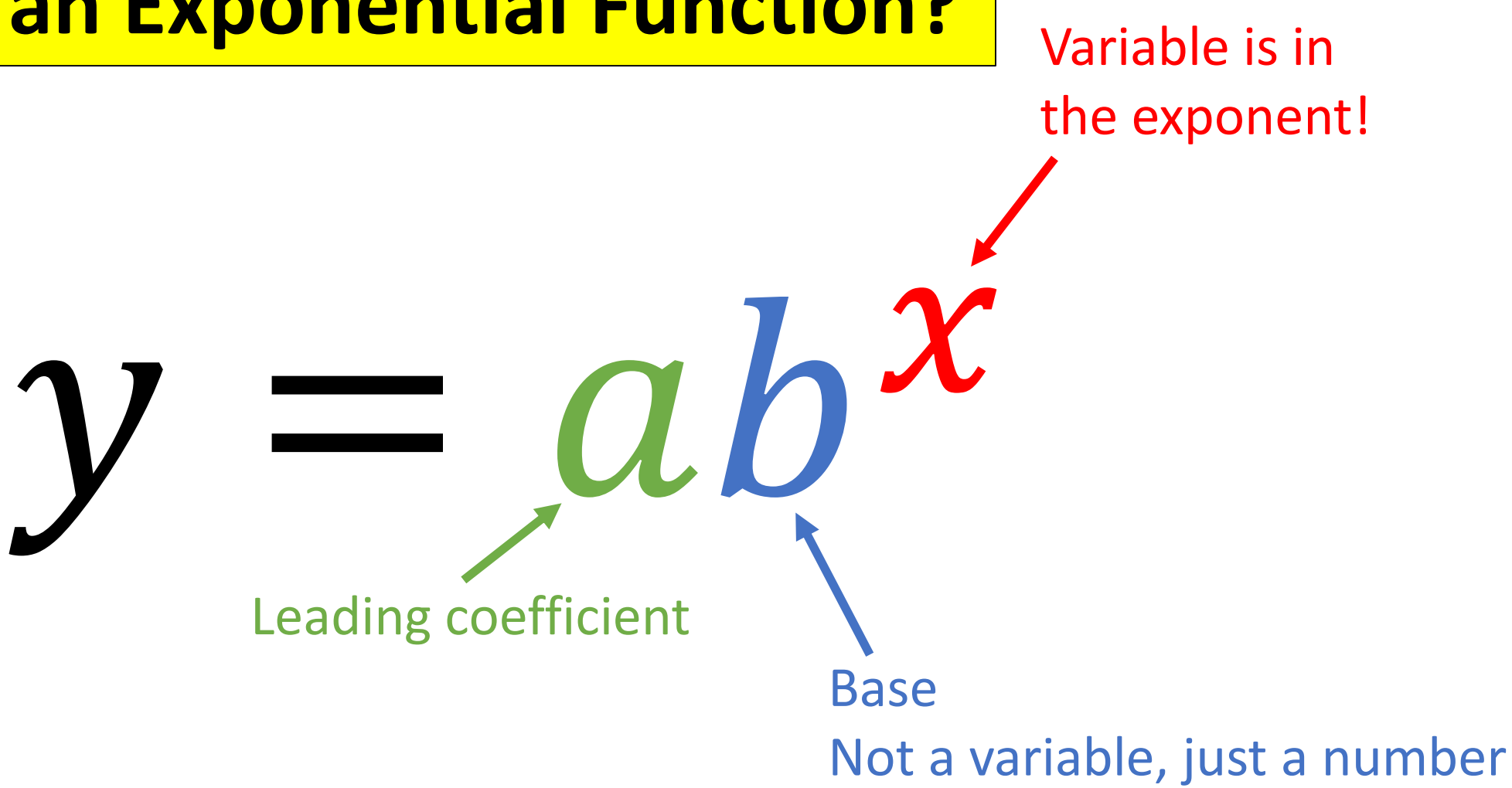
Core Vocabulary:

- Logarithm base b of y, p. 310
- Common logarithm, p. 311
- Natural logarithm, p. 311

Previous:

- Inverse functions

What is an Exponential Function?



Some are easy to solve!

- $4 = 2^x$ $\boldsymbol{x = 2}$
- $125 = 5^x$ $\boldsymbol{x = 3}$
- $\frac{1}{8} = 2^x$ $\boldsymbol{x = -3}$

Others, not so easy…

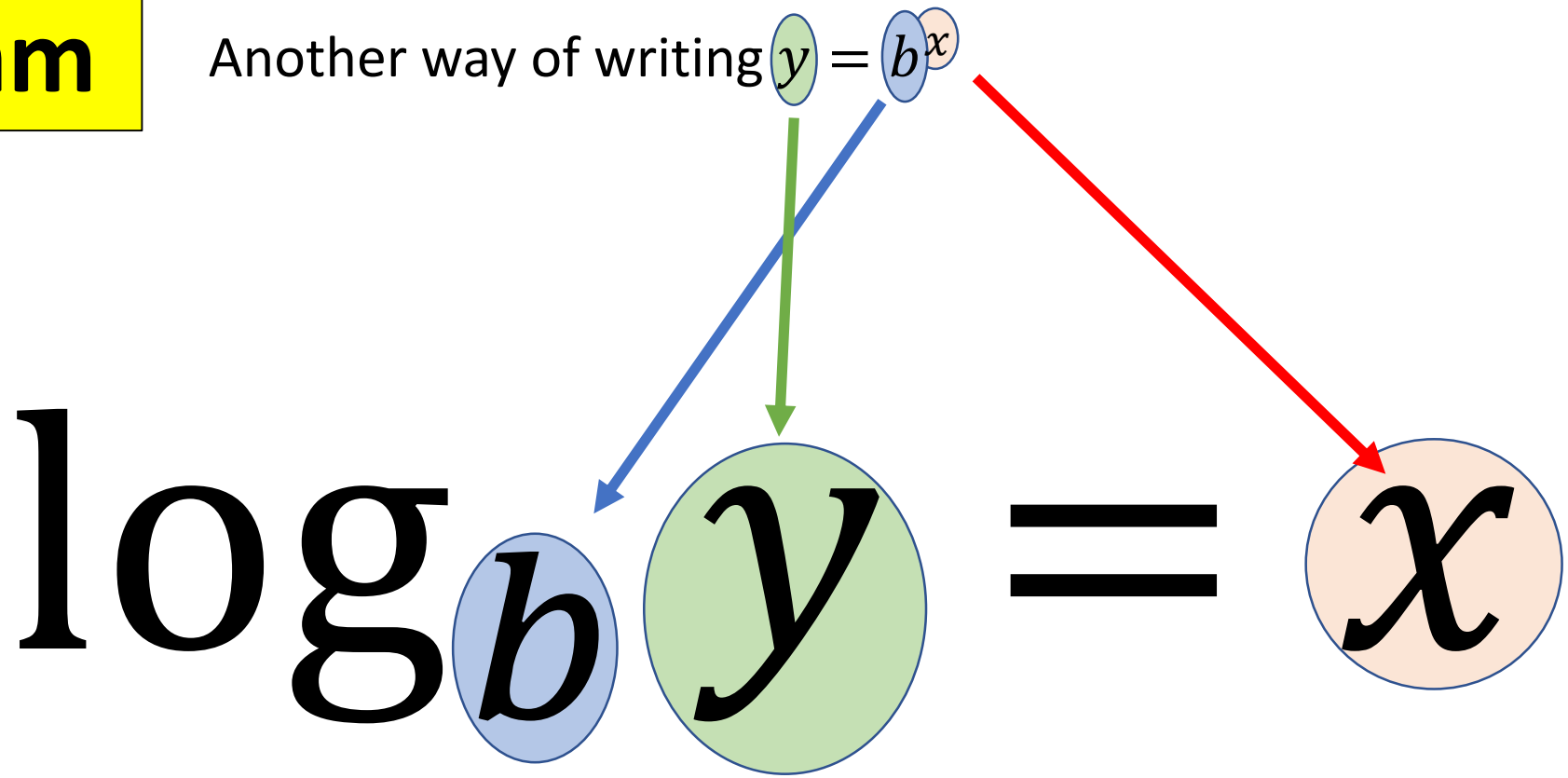
- $6 = 2^x$
- $13.01 = 5.3^x$

How do we solve these things when they get weird?

Logarithms! ☺

Logarithm

Another way of writing $y = b^x$



Read it as “Log base b of y is x”

…as an exponential function it is b to the x is y

Rewrite each equation in exponential form.

a. $\log_2 16 = 4$ **b.** $\log_4 1 = 0$ **c.** $\log_{12} 12 = 1$ **d.** $\log_{1/4} 4 = -1$

SOLUTION

Logarithmic Form	Exponential Form
a. $\log_2 16 = 4$	$2^4 = 16$
b. $\log_4 1 = 0$	$4^0 = 1$
c. $\log_{12} 12 = 1$	$12^1 = 12$
d. $\log_{1/4} 4 = -1$	$\left(\frac{1}{4}\right)^{-1} = 4$

Rewrite each equation in logarithmic form.

a. $5^2 = 25$ **b.** $10^{-1} = 0.1$ **c.** $8^{2/3} = 4$ **d.** $6^{-3} = \frac{1}{216}$

SOLUTION

Exponential Form	Logarithmic Form
a. $5^2 = 25$	$\log_5 25 = 2$
b. $10^{-1} = 0.1$	$\log_{10} 0.1 = -1$
c. $8^{2/3} = 4$	$\log_8 4 = \frac{2}{3}$
d. $6^{-3} = \frac{1}{216}$	$\log_6 \frac{1}{216} = -3$

Some special logarithm values:

- $\log_b 1 = ?$ $\longrightarrow$ $b^0 = 1$
  - $b^? = 1$ Rewrite as exponential
  - $? = 0$ …anything raised to the zero power is 1

- $\log_b b = ?$ $\longrightarrow$ $b^1 = b$
  - $b^? = b$ Rewrite as exponential
  - $? = 1$ …anything raised to the 1st power is itself

Evaluate each logarithm. Some easier ones you can maybe do in your head…

a. $\log_4 64$ **b.** $\log_5 0.2$ **c.** $\log_{1/5} 125$ **d.** $\log_{36} 6$

SOLUTION

To help you find the value of $\log_b y$, ask yourself what power of b gives you y.

a. What power of 4 gives you 64? $4^3 = 64$

b. What power of 5 gives you 0.2? $5^{-1} = 0.2$

c. What power of $\frac{1}{5}$ gives you 125? $\left(\frac{1}{5}\right)^{-3} = 125$

d. What power of 36 gives you 6? $36^{1/2} = 6$

Now … what about solving something like $6 = 2^x$? Not doing that in our heads! Gonna need our calculators for this one.

Look at your calculator, there are two buttons on it related to logarithms, can you find them?

- LOG
- LN

For each of these, what is the 2nd function (in blue)?

- LOG → 10^x
- LN → e^x

These are logarithms each with a specific base…

Common Logarithm:

- A logarithm with base 10
- Denoted as $\log_{10}$ … or as just $\log$
- If you see the word “log” with no base identified, it is automatically base 10

Natural Logarithm:

- A logarithm with base e
- Denoted as $\ln$ … can be written as $\log_e$ but you will rarely if ever see this
- If you see the word “ln”, it is automatically base e

This means the only exponential functions our calculator can directly help us with are ones with base 10 or base e.

So how do you solve an exponential function that does not have a base of 10 or e?

We will learn how to do that in a few days! Be patient!

Right now we are going to focus on common logs (base 10) and natural logs (base e).

Evaluate (a) log 8 and (b) ln 0.3 using a calculator. Round your answer to three decimal places.

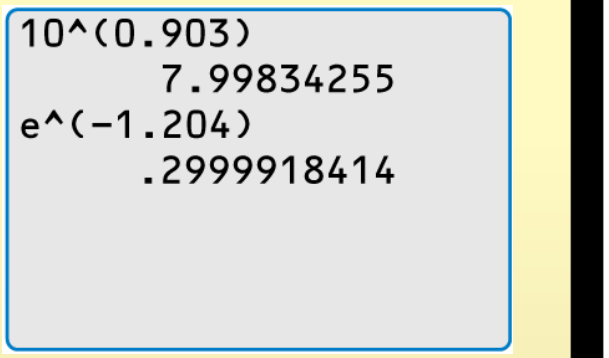
SOLUTION

Most calculators have keys for evaluating common and natural logarithms.

a. $\log 8 \approx 0.903$

b. $\ln 0.3 \approx -1.204$

Check your answers by rewriting each logarithm in exponential form and evaluating.



Check

Homework

Pg 314, #1-34